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Sur la Réduction de la Bisimulation de CHOCS à la Bisimulation du π-calcul

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Abstract

CHOCS and π-calculus are two extensions of CCS where, respectively, processes and channels are transmissible values. In previous work we have proposed a formalization of the notion of bisimulation for CHOCS. In this paper we suggest a more effective way to reason about this notion by means of a translation of CHOCS into a variant of the π-calculus.

Résumé

CHOCS et le π-calcul sont deux extensions de CCS où, respectivement, processus et canaux sont des valeurs transmissibles. Dans un travail précédent nous avons proposé une formalisation de la notion de bisimulation pour CHOCS. Dans ce papier nous présentons une méthode plus efficace de raisonner sur cette notion à l'aide d'une traduction de CHOCS dans une variante du π-calcul.

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1. Introduction

CHOCS (Thomsen[89], but see also Nielsen[88], Boudol[89]) and π-calculus (Engberg&-Nielsen[86], Milner&al.[89]) are two extensions of CCS where, respectively, processes and channels can be exchanged along channels.

There is a natural translation of CHOCS into the π-calculus. The idea is to simulate the delivery of a process by sending the name of a special channel that works like an activator for the process to be sent. More precisely let us use a CSP notation for input and output: c!s.P is the process which sends the transmissible value s along the channel c and then becomes P, c?x.P is the process which along the channel c receives some transmissible value v and becomes P with v at the place of x. Also let us suppose 'ν' denotes restriction, '+' sum, '|' parallel composition, and 'ø' the *nil* process, with the standard CCS semantics.[2] As usual τ is an internal action which corresponds to synchronization. The duplicator (or replicator) 'δ' stands for an operator which, roughly, when applied to a process P provides an unbounded number of copies of P. Abbreviations: (i) if the transmitted or received values are irrelevant we write simply c!.P and c?.P; (ii) we often omit to write the nil process. Then we informally define (a fragment of) the translation < >: CHOCS→π-calculus as follows:

$$<c!P'.P> = \nu l.(c!l.\ (<P>\ |\ \delta(l?.<P'>)\ )\ \text{ for } l\notin P, P', \qquad <x> = x!$$

The remaining part is trivially defined as:

$$<ø> = ø, \qquad <P\,|\,P'> = <P>\,|\,<P'>, \qquad <P+P'> = <P>+<P'>$$
$$<\nu c.P> = \nu c.<P>, \qquad <c?x.P> = c?x.<P>$$

Our goal is to use this translation in order to reduce reasoning about CHOCS bisimulation to reasoning about some variant of π-calculus bisimulation. The distinctive advantage of π-calculus bisimulation is that its verification requires an analysis which is *local* to the processes we are considering; on the contrary CHOCS bisimulation, in its general definition, requires infinitely many checks on rather arbitrary processes. We delay an evaluation of what is achieved by our reduction to the conclusions, and next we move forward to describe its basic ideas.

Consider the following CHOCS processes:

$$P \equiv \nu c.(c!(a!)\ |\ c?x.(x+b!)) \qquad P' \equiv \nu c.(c!ø\ |\ c?x.(a!+b!))$$

A reasonable equivalence on CHOCS programs should equate P and P'. We now look for an equivalence on the π-calculus that equates the translations:

$$<P> = \tau.\nu l.(\delta(l?a!)\ |\ (l!+b!)\ ) \qquad <P'> = \tau.(a! + b!)\ .$$

Note that a setting in which <P> = $\tau.(\tau.a! + b!)$ will not do as $\tau.(\tau.a! + b!)$ and $\tau.(a! + b!)$ are not even *weakly bisimilar*. Hence the first idea is to consider the communication on the channel l as a special one. Observe that this communication has only the function of activating a process. In the following we will assume that this type of communication is represented by a special action 't'. Then we have:

$$<P> = \tau.(t.a! + b!).$$

[2] We assume some familiarity with the process calculi literature (see, e.g., Milner[89]).

It is clear that we have made no progress unless in defining the legal observations we treat t differently from τ ! The second idea is to consider only transitions of the form '$\mapsto t^* \mapsto \alpha$' for $\alpha \neq t$, i.e. an arbitrary number of t actions followed by a 'real' α action. The intuition is that the execution mechanism is such that once we engage in a t action we will proceed with internal t actions till an action distinct from t is performed.[3] Next we can build a notion of bisimulation, say $\sim_{\pi,t}$, over this lts. It turns out that in this way we obtain the desired equivalence:

$$<P> = \tau.(t.a! + b!) \sim_{\pi,t} \tau.(a! + b!) = <P'> .$$

Having sketched the initial ideas we can outline proofs and results. In first approximation we formalize a variant of the π-calculus with t actions and modified transitions. We then show for P, Q CHOCS processes:

$$P \sim Q \Leftrightarrow <P> \sim_{\pi,t} <Q>.$$

More precisely we introduce an intermediary CHOCS calculus with t actions and modified transitions. We then build over this lts two notions of sgb, say $\sim_{c,t}$ and $\sim_{o,t}$, and prove the following chain of equivalences: (w.l.o.g. P, Q have no free process variables):

$$P \sim Q \Leftrightarrow P \sim_{c,t} Q \Leftrightarrow P \sim_{o,t} Q \Leftrightarrow <P> \sim_{\pi,t} <Q>.$$

In comparing with previous work we can make the following considerations. The techniques needed to prove these equivalences are of the same nature of those presented in Thomsen[89(a)] and Amadio[92]. The advance here is that we get a crisper characterization of the properties of the translation '< >' which in turn provides a strategy to reason about CHOCS bisimulation via a reduction to π-calculus bisimulation. In this respect the results in Thomsen[89(a)] have two main limitations: (1) they are obtained for a variant of CHOCS calculus, say CHτ, whose bisimulation does not relate in an obvious way to the one of the original calculus, (2) the notion of bisimulation which is considered is too strong, for instance the following implication dramatically fails to be true: $<P> \sim <Q> \Rightarrow P \sim_{CH\tau} Q$. In Amadio[92] we have fixed problem (2) by introducing a weaker notion of bisimulation (which is recalled in section 2). Here we fix problem (1) by taking a 'dual approach', i.e. our strategy is to find a variant of the π-calculus which fits the original CHOCS calculus rather than modify CHOCS in order to fit its π-calculus translation.

The technical development of the paper proceeds as follows. In section 2, following Amadio[92], we introduce a *generic* labelled transition system (lts) and a notion of strong ground bisimulation (sgb) on top of it. CHOCS and π-calculus are obtained as particular instances of this calculus. In section 3 we describe a variant of CHOCS based on the idea of activation channel as discussed above. On top of the modified lts we build two sgbs: $\sim_{c,t}$ and $\sim_{o,t}$. We then prove the first two steps of our chain of equivalences, i.e. $P \sim Q \Leftrightarrow P \sim_{c,t} Q \Leftrightarrow P \sim_{o,t} Q$. In section 4 we describe a variant of the π-calculus also based on the notion of activation channel. We introduce the sgb $\sim_{\pi,t}$, and prove the last step: $P \sim_{o,t} Q \Leftrightarrow <P> \sim_{\pi,t} <Q>$. In section 5 we put together and analyse our results.

[3] It is perfectly irrelevant to our purposes to discuss if this is a 'realistic' assumption.

2. CHOCS and π-calculus

We follow Amadio[92] where a uniform notation for the description of calculi with *complex transmissible values* and *static scoping* is proposed. We will omit all proofs and discussions and just give the basic definitions.

Kinds: The expressions of the language are classified in three kinds (K): processes (Pr), channels (Ch), and transmissible values (Tv). Hence: $K ::= Pr \mid Ch \mid Tv$.

Language: Channels are names, here represented as variables. Processes may have a complex structure. Transmissible values can be of various kinds, namely values, channels, or processes.

Processes	$P ::= \emptyset \mid v \mid (P+P) \mid (P \mid P) \mid (\delta P) \mid (\nu c.P) \mid (c\,!\,s.P) \mid (c\,?\,v.\,P)$		
Variables	$v ::= x \mid y \mid \ldots$	Channels	$c ::= v$
Transm. Values	$s ::= v \mid c \mid P$	Generic Exp.	$exp ::= c \mid P \mid s$

Context: A context Γ is a possibly empty sequence of pairs variable-kind, where all variables are distinct. We explicitly represent a context as: $v_1\!: K_1, \ldots, v_n\!: K_n$ $(n \geq 0)$.

Kind Judgments: A kind judgment has the shape: $\Gamma \supset exp\!: K$, to be read as exp has kind K in the context Γ. A judgment is provable, and we write $\vdash \Gamma \supset exp\!: K$, if it can be derived in the following system ("$\Rightarrow$" separates hypotheses from conclusion).

[asmp]	$v\!: K \in \Gamma \;\Rightarrow\; \Gamma \supset v\!: K$
[ø]	$\Rightarrow\; \Gamma \supset \emptyset\!: Pr$
[+]	$\Gamma \supset P\!: Pr,\; \Gamma \supset Q\!: Pr \;\Rightarrow\; \Gamma \supset (P+Q)\!: Pr$
[|]	$\Gamma \supset P\!: Pr,\; \Gamma \supset Q\!: Pr \;\Rightarrow\; \Gamma \supset (P \mid Q)\!: Pr$
[δ]	$\Gamma \supset P\!: Pr \;\Rightarrow\; \Gamma \supset (\delta P)\!: Pr$
[ν]	$\Gamma, c\!: Ch \supset P\!: Pr \;\Rightarrow\; \Gamma \supset (\nu c.P)\!: Pr$
[!]	$\Gamma \supset c\!: Ch,\; \Gamma \supset s\!: Tv,\; \Gamma \supset P\!: Pr \;\Rightarrow\; \Gamma \supset (c!s.\, P)\!: Pr$
[?]	$\Gamma \supset c\!: Ch,\; \Gamma, v\!: Tv \supset P\!: Pr \;\Rightarrow\; \Gamma \supset (c?v.\, P)\!: Pr$

It is routine to show by induction on the length of the derivations that we can derive (at the meta-level) rules of context *exchange*, *weakening*, *remove* of variables not appearing in the process, as well as a *substitution* rule.

Actions: Processes are described by their ability to perform actions. Roughly such actions can be of three types: (i) an internal action (τ), (ii) an input communication along a given channel (c?), (iii) an output communication of a transmissible value along a given channel (c!s). Hence we have as actions (α): $\alpha ::= \tau \mid c? \mid c!s$.

Labelled Transition System: We define a formal system to derive judgments of the shape '$\Gamma \supset P \mapsto\alpha\, \Gamma' \supset P'$' that should be read as "from the process P in the context Γ we make an action α and we generate a process P' in the context Γ' ".[4]

(!) $$\frac{}{\Gamma \supset (c!s.\ P) \mapsto c!s\ \Gamma \supset P}$$

(?) $$\frac{}{\Gamma \supset (c?v.\ P) \mapsto c?\ \Gamma, v{:}Tv \supset P}$$

(+.left) $$\frac{\Gamma \supset P \mapsto\alpha\ \Gamma' \supset P'}{\Gamma \supset (P+Q) \mapsto\alpha\ \Gamma' \supset P'}$$

(| .left) $$\frac{\Gamma \supset P \mapsto\alpha\ \Gamma' \supset P'}{\Gamma \supset (P \mid Q) \mapsto\alpha\ \Gamma' \supset (P' \mid Q)}$$

(δ) $$\frac{\Gamma \supset P \mid P \mapsto\alpha\ \Gamma' \supset P'}{\Gamma \supset \delta P \mapsto\alpha\ \Gamma' \supset P' \mid \delta P}$$

(ν) $$\frac{\Gamma, c{:}\ Ch \supset P \mapsto\alpha\ \Gamma, c{:}\ Ch, \Gamma' \supset P' \qquad c \notin \alpha}{\Gamma \supset (\nu c.P) \mapsto\alpha\ \Gamma, \Gamma' \supset (\nu c.P')}$$

(ν.open) $$\frac{\Gamma, c{:}\ Ch \supset P \mapsto d!s\ \Gamma, c{:}\ Ch, \Gamma'' \supset P' \qquad c \in s,\ d \neq c}{\Gamma \supset (\nu c.P) \mapsto d!s\ \Gamma, c{:}\ Ch, \Gamma'' \supset P'}$$

(| .com!?) $$\frac{\Gamma \supset P \mapsto d!s\ \Gamma, \Gamma' \supset P' \qquad \Gamma \supset Q \mapsto d?\ \Gamma, v{:}Tv \supset Q'}{\Gamma \supset (P \mid Q) \mapsto\tau\ \Gamma \supset \nu\Gamma'.(P' \mid [s/v]Q')}$$

There are also symmetric rules (+.right), (+.left), and (| .com?!) which we will treat implicitly in the following. The rule (ν.open) removes the restriction "νc" from the process νc.P whenever such process sends a transmissible value s containing c. In the case of an internal communication (rule (| .com!?)) these channels are closed again.

Proposition *(basic facts about the lts)*

(1) If $\vdash \Gamma \supset P \mapsto\alpha\ \Gamma' \supset P'$ then $\Gamma \subseteq \Gamma'$, and moreover
 if $\alpha \equiv \tau$ then $\Gamma \equiv \Gamma'$.
 if $\alpha \equiv d?$ then $\Gamma, x{:}Tv \equiv \Gamma'$ for some x.
 if $\alpha \equiv d!s$ then $\Gamma, c_1{:}\ Ch, \ldots, c_m{:}\ Ch \equiv \Gamma'$, for some $m \geq 0, c_1, \ldots, c_m$.

(2) If $\vdash \Gamma \supset P{:}\ Pr$ and $\vdash \Gamma \supset P \mapsto\alpha\ \Gamma, \Gamma' \supset P'$ then
 (a) $\vdash \Gamma, \Gamma' \supset P'{:}\ Pr$, and (b) if $\alpha \equiv d!s$ then $\vdash \Gamma, \Gamma' \supset s{:}\ Tv$.

Next we define the notion of strong *ground* bisimulation (sgb). The adjective 'ground' here refers to the fact that all assumptions in the context are considered as constants. Let $Pr(\Gamma)$ be $\{P \mid \vdash \Gamma \supset P{:}\ Pr\}$, and let Cont be the set of contexts. Denote

[4] **Conventions:** In the expressions νc.P and d?v.P the variables c and v are bound by ν and ? respectively. We identify terms which differ only by their bound variables. We denote with [exp/v]exp' the substitution of exp for v in exp'. We write $c \in s$ if the channel c occurs free in the transmissible value s. We also write $c \notin \alpha$ if c is not free in the action α, i.e. $\alpha \equiv d? \Rightarrow d \neq c$ and $\alpha \equiv d!s \Rightarrow d \neq c$ and $c \notin s$. If $\Gamma \equiv c_1{:}\ Ch, \ldots, c_m{:}\ Ch\ (m \geq 0)$ then $\nu\Gamma \equiv \nu c_1 \ldots \nu c_m$. By $\Gamma \cap \Gamma' = \emptyset$ we mean that no variable occurs both in Γ and Γ'. For the sake of readability in the following proofs we will feel free to omit contexts, kinds and '$\vdash$'.

with S, T,... a family of binary relations indexed over Cont, such that:[5]

$S{:}Cont \rightarrow \cup_{\Gamma \in Cont} Pr(\Gamma)^2, \ \ S(\Gamma) \subseteq Pr(\Gamma)^2$, for any Γ.

If $(P, Q) \in S(\Gamma)$ then we write more suggestively: $\Gamma \supset PSQ$.

Definition *(strong ground bisimulation)*

A relation S is a sgb if $\Gamma \supset P\,S\,Q$ and for any $\vdash \Gamma \supset P \mapsto_{\alpha} \Gamma' \supset P'$ then:

(τ) $\alpha \equiv \tau$ implies there is $\vdash \Gamma \supset Q \mapsto_{\tau} \Gamma \supset Q'$, $\Gamma \supset P'SQ'$.

(?) $\alpha \equiv d?$, $\Gamma' \equiv \Gamma, x{:}Tv$ implies there is $\vdash \Gamma \supset Q \mapsto_{d?} \Gamma, x{:}Tv \supset Q'$,
any $\vdash \Gamma, \Gamma'' \supset s{:}Tv$, $\Gamma, \Gamma'' \supset [s/x]P'\ S\ [s/x]Q'$,

(!) $\alpha \equiv d!s$, $\Gamma' \equiv \Gamma, \Gamma_1$ implies there is $\vdash \Gamma \supset Q \mapsto_{d!s'} \Gamma, \Gamma_2 \supset Q'$,
any $\vdash \Gamma, \Gamma'', x{:}Tv \supset R{:}Pr$, with $\Gamma'' \cap \Gamma_1 = \emptyset$ and $\Gamma'' \cap \Gamma_2 = \emptyset$,
$\Gamma, \Gamma'' \supset \nu\Gamma_1.(P' \mid [s/x]R)\ S\ \nu\Gamma_2.(Q' \mid [s'/x]R)$.

and *symmetrically*.

Sgbs are closed under arbitrary (indexed) unions. We write $\Gamma \supset P{\sim}Q$ iff $\Gamma \supset PSQ$, for some sgb S. Apart for the standard monoidal laws of sum and parallel composition the following restriction commutations and duplication laws are of interest:

(ν.+)	$\Gamma \supset \nu c.P + \nu c.P' \sim \nu c.(P+P')$.	(ν.|)	$\Gamma \supset (\nu c.P) \mid P' \sim \nu c.(P \mid P')$, if $c \notin P'$
(ν.!)	$\Gamma \supset (d!s.\ \nu c.P) \sim \nu c.(d!s.\ P)$, if $c \notin s$	(ν.?)	$\Gamma \supset (d?v.\ \nu c.P) \sim \nu c.(d?v.\ P)$
(δ)	$\Gamma \supset \delta P \sim P \mid \delta P$	($\delta\delta$)	$\Gamma \supset \delta P \sim \delta(\delta P)$
(δ.|)	$\Gamma \supset \delta(P \mid Q) \sim \delta P \mid \delta Q$.		

Sgb treats all assumptions as constants. We may wish to extend the notion of bisimulation to processes containing free variables. We represent variables by underlying the corresponding assumptions in the context. By convention if Γ is a context, $\underline{\Gamma}$ is the same context where we underline all assumptions. We state that two processes are strongly bisimilar if they are strongly ground bisimilar for all possible ground instances of the variables.

Definition *(strong bisimulation)*

$\Gamma, \underline{x_1{:}K_1, \ldots, x_n{:}K_n} \supset P{\sim}Q$ if for any $\Gamma, \Gamma' \supset exp_i{:}K_n$ $(i=1,..,n)$,
$\Gamma, \Gamma' \supset [exp_1/x_1, \ldots, exp_n/x_n]P \sim [exp_1/x_1, \ldots, exp_n/x_n]Q$.

Representation of CHOCS and π-calculus

We represent CHOCS processes by supposing that the kinds of transmissible values and processes coincide: Tv≡Pr.

We represent π-calculus processes by supposing that the kinds of transmissible values and channels coincide, Tv≡Ch, and moreover that variables in the context are not of the process kind. Concerning the lts the major simplification which can be carried on consists in observing that:

$$\vdash \Gamma \supset P \mapsto_{d!c} \Gamma, \Gamma' \supset P' \quad \text{implies} \quad \Gamma' \equiv \emptyset \ \text{ or } \ \Gamma' \equiv c{:}Ch\ .$$

[5] In the following we will refer to S, T,... simply as relations.

An important fact is that the (!) clause of the definition of sgb can be simplified as follows:

$(!\pi)$ $\alpha \equiv d!c$, $\Gamma' \equiv \Gamma,\Gamma_1$ implies there is
$\vdash \Gamma \supset Q \mapsto^{d!c} \Gamma,\Gamma_1 \supset Q'$, $\Gamma,\Gamma_1 \supset P' \, S \, Q'$.

This result together with the simple remark that in the input clause only a finite number of inputs needs to be considered supports the view that bisimulation in the π-calculus can be checked in an essentially 'local' way (see Amadio[92] for details).[6] □

3. The CH$_t$-calculus

We split the collection of channel names in two disjoint classes:

- *standard* channels denoted with: c, c',...
- *activation* channels denoted with: l, l',...

In the following x, y will denote ambiguously standard and activation channels. When we create a channel with the operator ν we create either a standard channel, say νc, or an activation channel, say νl. The grammar of *CH$_t$-processes* is:

$$P ::= \emptyset \mid v \mid l! \mid (P+P) \mid (P \mid P) \mid (\delta P) \mid (\nu c.P) \mid (\nu l.P) \mid (c\,!P.P) \mid (c\,?\,v.\,P) \mid l?.P$$

The rules to derive kind judgments are immediately extended. Note that the action l! cannot be prefixed to another process, i.e. l! can only occur where a variable could (this is as much as we need). We now have six types of actions (α):

$$\alpha ::= c! \mid c? \mid \tau \mid l! \mid l? \mid t$$

Note that in the output action along a standard channel (c!) we drop the transmitted process. The t action stands for an internal action which represents the synchronization on an activation channel.

Lts for CH$_t$

We keep the following rules of the lts in section 2: (?), (+.left/right), (|.left/right), (δ), (ν), We have the following obvious specializations of the rules (!), (?), (ν) and (|.com!?) to deal with activation channels:

$(!)_t$ $$\frac{}{\Gamma \supset l! \mapsto^{l!} \Gamma \supset \emptyset}$$

$(?)_t$ $$\frac{}{\Gamma \supset (l?.\,P) \mapsto^{l?} \Gamma \supset P}$$

$(\nu)_t$ $$\frac{\Gamma,l{:}\,Ch \supset P \mapsto^{\alpha} \Gamma,l{:}\,Ch,\Gamma' \supset P' \qquad l \notin \alpha}{\Gamma \supset (\nu l.P) \mapsto^{\alpha} \Gamma,\Gamma' \supset (\nu l.P')}$$

$(|.com!?)_t$ $$\frac{\Gamma \supset P \mapsto^{l!} \Gamma \supset P' \qquad \Gamma \supset Q \mapsto^{l?} \Gamma \supset Q'}{\Gamma \supset (P \mid Q) \mapsto^{t} \Gamma \supset (P' \mid Q')}$$

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[6] In the terminology of Milner&al[89] the input clause of sgb is based on the idea of 'late binding'. 'Early' and 'late' binding induce different bisimulations on π-calculus processes. However the translations of CHOCS processes in π-calculus are not sensitive to the differences between early and late bisimulations.

On the other hand the rules (!) and (| .com!?) become:

$$(!) \qquad \frac{}{\Gamma \supset c!P'.\,P \mapsto^{c!} \Gamma, l{:}Ch \supset \delta(l?P') \mid P} \qquad (l \notin \Gamma)$$

$$(\mid.com!?) \qquad \frac{\Gamma \supset P \mapsto^{c!} \Gamma, l{:}Ch \supset P' \qquad \Gamma \supset Q \mapsto^{c?} \Gamma, x{:}Pr \supset Q'}{\Gamma \supset (P \mid Q) \mapsto^{\tau} \Gamma \supset \nu l.(P' \mid [l!/x]Q')}$$

The (!) rule mimicks the < >-translation. Rather than sending the process P' it makes visible an activator for P'. Note that because of the form of (!) the rule (ν.open) is now superfluous. We identify CH processes with the subset of CH_t-processes without activation channels. However the analysis of the semantic properties of this embedding requires some care as we will see next.

Derived lts system

In the first place from the lts '$\mapsto$' we derive a new lts '$\twoheadrightarrow$' as follows:

$$\Gamma \supset P \twoheadrightarrow^{\alpha} \Gamma' \supset P' \quad \text{iff} \quad \Gamma \supset P \mapsto^{t} \Gamma \supset P_1 \ldots \mapsto^{t} \Gamma \supset P_n \mapsto^{\alpha} \Gamma' \supset P' \qquad (\alpha \neq t,\ n \geq 0)$$

The intuition is that a process can be observed only after having performed a 'real' action α, and not while performing internal activations of processes. It is a useful exercise to redefine '$\twoheadrightarrow$' inductively on the structure of the process (look in particular at the rules for parallel composition).

Two variants of sgb for CH_t

On top of the lts '$\mapsto$' and '$\twoheadrightarrow$' for CH_t we build two notions of sgb.

Definition *(notions of sgbs for CH_t)*

A relation S is a *c-sgb* (*o-sgb*) if $\Gamma \supset P\,S\,Q$ and for any $\vdash \Gamma \supset P \mapsto^{\alpha} \Gamma' \supset P'$ then:

$\alpha \equiv \tau$, t, l?, or l! implies there is $\vdash \Gamma \supset Q \mapsto^{\alpha} \Gamma \supset Q'$, $\Gamma \supset P'SQ'$.

$\alpha \equiv c?$, $\Gamma' \equiv \Gamma, x{:}Pr$ implies there is $\vdash \Gamma \supset Q \mapsto^{c?} \Gamma, x{:}Pr \supset Q'$,
any $\vdash \Gamma, \Gamma'' \supset R{:}\ Pr$ *CH-process*, $\Gamma, \Gamma'' \supset [R/x]P'\ S\ [R/x]Q'$,

$\alpha \equiv c!$, $\Gamma' \equiv \Gamma, l{:}Ch$ implies there is $\vdash \Gamma \supset Q \mapsto^{c!} \Gamma, l{:}Ch \supset Q'$ such that:

c-sgb any $\vdash \Gamma, \Gamma'', x{:}Pr \supset R{:}Pr$, *CH-process* $\Gamma, \Gamma'' \supset \nu l.(P' \mid [l!/x]R)\ S\ \nu l.(Q' \mid [l!/x]R)$.
(*o-sgb* $\Gamma, l{:}Ch \supset P'SQ'$).

and symmetrically. We write $\Gamma \supset P \sim_c Q$ ($\Gamma \supset P \sim_o Q$) if $\Gamma \supset P\,S\,Q$ for some S c-sgb (o-sgb). If we substitute everywhere the relation $\mapsto$ with the relation $\twoheadrightarrow$ we obtain the notion of ct-sgb (ot-sgb). The greatest of these sgbs is denoted with $\sim_{ct}$ ($\sim_{ot}$).

We will show that closed and open conditions in the c! clause of sgb definition lead to the *same* equivalence. First observe the following easy fact:

If $\Gamma \supset P \sim_{\theta} Q$ then $\Gamma \supset P \sim_{\theta t} Q$, for $\theta ::= c \mid o$.

The vice versa obviously fails, e.g. $t.a! \sim_{\theta t} a!$ but *not* $t.a! \sim_{\theta} a!$. A more interesting observation is that also $t.a!+b! \sim_{\theta t} a!+b!$. This emphasizes that t-sgb is not quite like weak bisimulation. As a matter of fact t-sgbs can be regarded as strong bisimulations over a distinct labelled transition system. It can be shown that they satisfy the standard monoidal, restriction, and commutation laws discussed in section 2. Moreover t-sgbs satisfy the standard congruence rules of sgb, e.g.,

$$\Gamma \supset P \sim_{\theta t} Q \ , \ \Gamma \supset P' \sim_{\theta t} Q' \ \Rightarrow \ \Gamma \supset P+P' \sim_{\theta t} Q+Q', \ \ \Gamma \supset P \mid P' \sim_{\theta t} Q \mid Q' .$$

This is not completely trivial because, as mentioned above, the rules to derive transitions of parallel compositions are not those of section 2.

Simulating Substitution

Always following the spirit of the translation we may think to simulate the notion of substitution via activation channels as follows:

$$\{P/x\}R \equiv \nu l.([l!/x]R \mid \delta(l?.P)) \qquad \text{(for } l \notin P, R)$$

It is probably fair to say that the notions of t-sgbs are invented in order to make equivalent the actual substitution with its simulation as stated in the following

Lemma *(Substitution)*

Suppose $\vdash \Gamma, \Gamma'', x{:}Pr \supset R{:}Pr$, $\vdash \Gamma, \Gamma' \supset P{:}Pr$, where $\Gamma' \cap \Gamma'' = \emptyset$, and P, R are CH_t-processes. Then: $\underline{\Gamma, \Gamma', \Gamma''} \supset [P/x]R \sim_{\theta t} \{P/x\}R$, for $\theta ::= c \mid o$.

Proof

By induction on the length of R kind judgment. The crucial case arising for $R \equiv x$ gives $P \sim_{\theta_t} t.P$. Note that this would hold for weak bisimulation, with τ at the place of t, but the basic problem is that this equivalence is not a congruence and so we fall into the counter-example mentioned in the introduction. In the following we only consider the most important cases:

[asmp] If $R \equiv x$ we use the equivalence: $\Gamma \supset P \sim_{\theta_t} t.P$.

[|] We use the following equivalence:

$$\nu l.(\, [l!/x]P \mid [l!/x]P' \mid \delta(l?Q)\,) \sim_{\theta t} \nu l.(\, [l!/x]P \mid \delta(l?Q)\,) \mid \nu l.(\, [l!/x]P' \mid \delta(l?Q)\,)$$

[!] $[P/x](c!R.\, R') = c![P/x]R.[P/x]R' \longmapsto^{c!} l \supset (\delta(l?.[P/x]R) \mid [P/x]R') \sim_{\theta t}$
$\delta(l?.\{P/x\}R) \mid \{P/x\}R' \sim_{\theta t} \nu l'.(\, \delta(l?.[l'!/x]R) \mid \delta(l'?.\, P)\,)) \mid \nu l'.([l'!/x]R' \mid \delta(l'?.\, P)\,) \sim_{\theta t}$
$\nu l'.(\, \delta(l?.[l'!/x]R) \mid [l'!/x]R' \mid \delta(l'?.\, P)\,) \sim_{\theta t} \{P/x\}(c!R.\, R')$.

[?] $\{P/x\}(c?v.\, R) \longmapsto^{c?} v \supset \{P/x\}R$, $[P/x](c?v.\, R) \longmapsto^{c?} v \supset [P/x]R$, and by ind. hyp.
$\underline{v} \supset [P/x]R \sim_{\theta t} \{P/x\}R$. $\square$

Simplifying the Input Clause

The following lemma together with the substitution lemma is the key to a drastic simplification of the input clause of sgb.

Lemma *(Input)*

Suppose $\vdash \Gamma, x{:}Pr \supset P{:}Pr$, $\vdash \Gamma, x{:}Pr \supset Q{:}Pr$, $\theta ::= c \mid o$. Then any $\vdash \Gamma, \Gamma'' \supset R{:}Pr$ CH-process $\Gamma, \Gamma'' \supset [R/x]P \sim_{\theta_t} [R/x]Q$ implies $\vdash \Gamma, l{:}Ch \supset [l!/x]P \sim_{\theta_t} [l!/x]Q$.

Proof

Observe that if in $[l!/x]P$, l is fresh then $[l!/x]P$ derivatives can only perform transitions affecting $l!$ if the relative action is $l!$. This is because in the (?) clause of sgb we only need to consider processes in CH. On the contrary in $[c!/x]P$, c fresh, c can be affected by later inputs. This motivates the following definition

$$\Gamma,l{:}Ch\supset [l!/x]P \; S \; [l!/x]Q \quad \text{if } \exists \{c_i\}_{i<\omega} \text{ infinite } \Gamma,c_i{:}Ch\supset [c_i!/x]P \sim_{\theta t} [c_i!/x]Q.$$

One shows that S is a θt-sgb. We will only consider the input case (c?):

(i) Suppose: $l\supset [l!/x]P \; S \; [l!/x]Q$, $[l!/x]P \twoheadrightarrow^{c?} y\supset [l!/x]P'$.

(ii) Then: $[c_i!/x]P \sim_{\theta t} [c_i!/x]Q$, $[c_i!/x]P \twoheadrightarrow^{c?} y\supset [c_i!/x]P'$. Hence:
$[c_i!/x]Q \twoheadrightarrow^{c?} y\supset [c_i!/x]Q'$, any R in CH $[R/y][c_i!/x]P' \sim_{\theta t} [R/y][c_i!/x]Q'$.
Note that for any given R there are infinitely many fresh c_i which satisfy this equivalence.

(iii) So: $[l!/x]Q \twoheadrightarrow^{c?} y\supset [l!/x]Q'$, any R in CH $[R/y][c_i!/x]P' \; S \; [R/y][c_i!/x]Q'$. □

Proviso

The structure of the previous proof intended as: (a) definition of a relation S in terms of a sgb, (b) analysis of the possible actions, (c) back and forth between S and the sgb divided in steps (i)-(iii), will be repeated ad nauseam in the following without further comments.

Proposition *(generic input)* [7]

Suppose $\vdash\Gamma,x{:}Pr\supset P{:}Pr$, $\vdash\Gamma,x{:}Pr\supset Q{:}Pr$, $\theta ::= c \mid o$. Then any $\vdash\Gamma,\Gamma''\supset R{:}Pr$ CH-process $\Gamma,\Gamma''\supset [R/x]P \sim_{\theta t} [R/x]Q$ iff $\vdash\Gamma,l{:}Ch\supset [l!/x]P \sim_{\theta t} [l!/x]Q$ (l fresh).

Proof

Direction ($\Rightarrow$) follows by the previous lemma. For direction ($\Leftarrow$) observe:

$$[l!/x]P \sim_{\theta t} [l!/x]Q \;\Rightarrow\; [l!/x]P \mid \delta(l?.R) \sim_{\theta t} [l!/x]Q \mid \delta(l?.R) \;\Rightarrow$$
$$\{R/x\}P \sim_{\theta t} \{R/x\}Q \;\Rightarrow\; [R/x]P \sim_{\theta t} [R/x]Q\,. \qquad \square$$

CH actions vs. CH_t-actions

Using the substitution lemma and induction on the length of derivations one can relate CH and CH_t-actions of a CH process P as follows ($\theta ::= c \mid o$):

From CH	*to* CH_t
$\vdash\Gamma\supset P \mapsto^{c!P_1} \Gamma,\Gamma'\supset P'$	$\vdash\Gamma\supset P \mapsto^{c!} \Gamma,l{:}Ch\supset P'' \;\wedge\; \Gamma,l{:}Ch\supset P'' \sim_{\theta t} \nu\Gamma'.(\delta(l?P_1) \mid P')$.
$\vdash\Gamma\supset P \mapsto^{c?} \Gamma,x{:}Pr\supset P'$	$\vdash\Gamma\supset P \mapsto^{c?} \Gamma,x{:}Pr\supset P'$.
$\vdash\Gamma\supset P \mapsto^{\tau} \Gamma\supset P'$	$\vdash\Gamma\supset P \mapsto^{\tau} \Gamma\supset P'' \;\wedge\; \Gamma\supset P' \sim_{\theta t} P''$.

From CH_t	*to CH*
$\vdash\Gamma\supset P \mapsto^{c!} \Gamma,l{:}Ch\supset P''$	$\vdash\Gamma\supset P \mapsto^{c!P_1} \Gamma,\Gamma'\supset P' \;\wedge\; \Gamma,l{:}Ch\supset P'' \sim_{\theta t} \nu\Gamma'.(\delta(l?P_1) \mid P')$.
$\vdash\Gamma\supset P \mapsto^{c?} \Gamma,x{:}Pr\supset P'$	$\vdash\Gamma\supset P \mapsto^{c?} \Gamma,x{:}Pr\supset P'$.
$\vdash\Gamma\supset P \mapsto^{\tau} \Gamma\supset P''$	$\vdash\Gamma\supset P \mapsto^{\tau} \Gamma\supset P' \;\wedge\; \Gamma\supset P' \sim_{\theta t} P''$.

[7] This is in contrast with what happens with the π-calculus where several alternatives *have* to be checked (see Amadio[92], section 5 for a discussion).

The following proposition is the first step in the reduction of CHOCS sgb to π-calculus sgb.

Proposition *(reducing CHOCS sgb to ct-sgb)*

Suppose P and Q are CH-processes in the context Γ. Then

$\Gamma \supset P \sim Q \iff \Gamma \supset P \sim_{c,t} Q$

Proof

($\Leftarrow$) We define $\Gamma \supset P\, S\, Q$ if $\Gamma \supset P \sim_{c,t} Q$ and show that S is a sgb. We use the correspondences between the actions.

(!) (i) $P\ S\ Q,\ P \mapsto^{c!} P_1\ \Gamma' \supset P'$

(ii) $P \sim_{c,t} Q,\ P \twoheadrightarrow^{c!}\ l \supset P'',\ l \supset P'' \sim_{c,t} \nu\Gamma'.(\delta(l?P_1) \mid P'),$
$Q \twoheadrightarrow^{c!}\ l \supset Q'',\ \nu l.(P'' \mid [l!/x]R) \sim_{c,t} \nu l.(Q'' \mid [l!/x]R)$, any R in CH,
$\nu l.(P'' \mid [l!/x]R) \sim_{c,t} \nu l.(\nu\Gamma'.(\delta(l?P_1) \mid P') \mid [l!/x]R) \sim_{c,t}$
$\nu\Gamma'.(P' \mid \{P_1/x\}R) \sim_{c,t} \nu\Gamma'.(P' \mid [P_1/x]R).$

(iii) $Q \mapsto^{c!} Q_1\ \Gamma'' \supset Q',\ \nu l.(Q'' \mid [l!/x]R) \sim_{c,t} \nu\Gamma''.(Q' \mid [Q_1/x]R),$
$\nu\Gamma'.(P' \mid [P_1/x]R)\ S\ \nu\Gamma''.(Q' \mid [Q_1/x]R)$, any R in CH.

(τ) (i) $P\ S\ Q,\ P \mapsto^{\tau} P'$

(ii) $P \sim_{c,t} Q,\ P \twoheadrightarrow^{\tau} P'',\ P'' \sim_{c,t} P',\ Q \twoheadrightarrow^{\tau} Q'',\ P'' \sim_{c,t} Q''$

(iii) $Q \rightarrow^{\tau} Q',\ Q' \sim_{c,t} Q'',\ P'\ S\ Q'$.

(?) (i) $P\ S\ Q,\ P \mapsto^{c?}\ x \supset P'$

(ii) $P \sim_{c,t} Q,\ P \twoheadrightarrow^{c?}\ x \supset P',$
$Q \twoheadrightarrow^{c?}\ x \supset Q',\ [R/x]P' \sim_{c,t} [R/x]Q'$, any R in CH.

(iii) $Q \mapsto^{c?}\ x \supset Q',\ [R/x]P'\ S\ [R/x]Q'$, any R in CH.

($\Rightarrow$) We define $\Gamma \supset P\, S\, Q$ if $\Gamma \supset P' \sim Q',\ \Gamma \supset P' \sim_{c,t} P,\ \Gamma \supset Q' \sim_{c,t} Q$ and show that S is a ct-sgb. This derives from the observation that if we take P, Q in CH then the check '$P \sim_{c,t} Q$' can be reduced, up to '$\sim_{c,t}$', to the check of processes in CH. The proof is now similar to the previous one. Observe that it is important that, in the (!) and (?) clauses of ct-sgb definition, we suppose R ranging over CH processes. □

Example

Consider the problem: Γ, <u>x:Pr, y:Pr</u>$\supset P \sim Q$. Clearly this can be reduced to: Γ, c:Ch $\supset c?x.c?y.P \sim c?x.c?y.Q$. If we now consider the definition of sgb for CH (section 2) we see that we should verify $\Gamma, \Gamma'' \supset [R/x, R'/y]P \sim [R/x, R'/y]Q$, for all suitable R, R', Γ''. By the previous proposition we could equivalently consider the problem: Γ, c:Ch$\supset c?x.c?y.P \sim_{c,t} c?x.c?y.Q$. But now we can apply the (generic input) proposition and reduce to: Γ, l:Ch, l':Ch$\supset [l!/x,l'!/y]P \sim_{c,t} [l!/x,l'!/y]Q$, which is quite more manageable ! □

The second step of the reduction consists essentially in showing that the ct-clause in the input case of the definition of sgb for CH_t is not weaker than the ot-clause.

Proposition *(The equivalence of ct-sgb and ot-sgb)*

Suppose P and Q are CH_t-processes in the context Γ. Then

$\Gamma \supset P \sim_{c,t} Q \Leftrightarrow \Gamma \supset P \sim_{o,t} Q$.

Proof

($\Leftarrow$) In order to show that $\sim_{o,t}$ is a ct-sgb the basic observation is:

$\Gamma, l{:}Ch \supset P \sim_{o,t} Q \Rightarrow \Gamma,\Gamma'' \supset \nu l.(P \mid [l!/x]R) \sim_{o,t} \nu l.(Q \mid [l!/x]R)$, any suitable R.

($\Rightarrow$) Suppose given for any $(l_1, ..., l_n)$ an infinite family $\{(c_{1,i}, ..., c_{n,i})\}_{i<\omega}$ such that: (a) for any j $\{c_{j,i}\}_{i<\omega}$ is infinite, (b) for any i, $j \neq j' \Rightarrow c_{i,j} \neq c_{i,j'}$. Here, as in the input lemma, we need an infinite family to avoid potential clashes in the input clause. We write $C[P_1,..., P_n]$ for a process in which *may* occur as subterms the processes $P_1,..., P_n$. We define a relation S as follows and we show S is an ot-sgb:

$\Gamma, l_1, ..., l_n \supset C[\delta(l_1?P_1),..., \delta(l_n?P_n)]\ S\ D[\delta(l_1?Q_1),..., \delta(l_n?Q_n)]$ if
$\Gamma,\Gamma' \supset \nu l_1...\nu l_n.(C[\delta(l_1?P_1),..., \delta(l_n?P_n)] \mid \delta(c_{1,i}?.l_1!) \mid ... \mid \delta(c_{n,i}?.l_n!)) \sim_{c,t}$
$\nu l_1...\nu l_n.(D[\delta(l_1?Q_1),..., \delta(l_n?Q_n)] \mid \delta(c_{1,i}?.l_1!) \mid ... \mid \delta(c_{n,i}?.l_n!))$
for infinitely many i and suitable Γ'.

with the conventions that: (a) l_i and $c_{j,i}$ only occur in the displayed positions, (b) $c_{j,i}$ is always understood as being free, as well as l_i unless we explicitly write a binder νl_i. The interesting cases arise for the actions: c!, l_i?, c? . In the following we will treat the case n=1, the need to generalize to n appears in the (c!) clause.

(c!) (i) $l_1 \supset C[\delta(l_1?P_1)] \twoheadrightarrow^{c!} l_1,l_2 \supset C'[\delta(l_1?P_1), \delta(l_2?P_2)]$.

(ii) $\nu l_1.(C[\delta(l_1?P_1)] \mid \delta(c_{1,i}?.l_1!)) \twoheadrightarrow^{c!} l_2 \supset \nu l_1.(C'[\delta(l_1?P_1), \delta(l_2?P_2)] \mid \delta(c_{1,i}?.l_1!))$,
$\nu l_1.(D[\delta(l_1?Q_1)] \mid \delta(c_{1,i}?.l_1!)) \twoheadrightarrow^{c!} l_2 \supset \nu l_1.(D'[\delta(l_1?Q_1), \delta(l_2?Q_2)] \mid \delta(c_{1,i}?.l_1!))$,
$\nu l_2.\nu l_1.(C'[\delta(l_1?P_1), \delta(l_2?P_2)] \mid \delta(c_{1,i}?.l_1!) \mid \delta(c_{2,i}?.l_2!)) \sim_{c,t}$
$\nu l.\nu l_1.(D'[\delta(l_1?Q_1), \delta(l_2?Q_2)] \mid \delta(c_{1,i}?.l_1!) \mid \delta(c_{2,i}?.l_2!))$,
by taking $R(x) \equiv \delta(c_{2,i}?.x)$.

(iii) $l_1 \supset D[\delta(l_1?Q_1)] \twoheadrightarrow^{c!} l_1,l_2 \supset D'[\delta(l_1?Q_1), \delta(l_2?Q_2)]$,
$l_1,l_2 \supset C'[\delta(l_1?P_1), \delta(l_2?P_2)]\ S\ D'[\delta(l_1?Q_1), \delta(l_2?Q_2)]$.

(l_1?) (i) $l_1 \supset C[\delta(l_1?P_1)] \twoheadrightarrow^{l_1?} l_1 \supset C'[\delta(l_1?P_1)]$.

(ii) $\nu l_1.(C[\delta(l_1?P_1)] \mid \delta(c_{1,i}?.l_1!)) \twoheadrightarrow^{c_{1,i}?} \sim_{c,t} \nu l_1.(C''[\delta(l_1?P_1)] \mid \delta(c_{1,i}?.l_1!) \mid l_1!)$
$\sim_{c,t} \nu l_1.(C'[\delta(l_1?P_1)] \mid \delta(c_{1,i}?.l_1!))$. Hence:
$\nu l_1.(D[\delta(l_1?Q_1)] \mid \delta(c_{1,i}?.l_1!)) \twoheadrightarrow^{c_{1,i}?} \sim_{c,t} \nu l_1.(D''[\delta(l_1?Q_1)] \mid \delta(c_{1,i}?.l_1!) \mid l_1!)$
$\sim_{c,t} \nu l_1.(D'[\delta(l_1?P_1)] \mid \delta(c_{1,i}?.l_1!))$, for a suitable D'[_], and
$\nu l_1.(C'[\delta(l_1?P_1)] \mid \delta(c_{1,i}?.l_1!)) \sim_{c,t} \nu l_1.(D'[\delta(l_1?P_1)] \mid \delta(c_{1,i}?.l_1!))$.

(iii) $l_1 \supset D[\delta(l_1?Q_1)] \twoheadrightarrow^{l_1?} l_1 \supset D'[\delta(l_1?Q_1)]$,
$l_1 \supset C'[\delta(l_1?P_1)]\ S\ D'[\delta(l_1?Q_1)]$.

(c?) (i) $l_1 \supset C[\delta(l_1?P_1)] \twoheadrightarrow^{c?} l_1,x \supset C'[\delta(l_1?P_1), x]$.

(ii) $\nu l_1.(C[\delta(l_1?P_1)] \mid \delta(c_{1,i}?.l_1!)) \twoheadrightarrow^{c?} x \supset \nu l_1.(C'[\delta(l_1?P_1), x] \mid \delta(c_{1,i}?.l_1!))$,
$\nu l_1.(D[\delta(l_1?Q_1)] \mid \delta(c_{1,i}?.l_1!)) \twoheadrightarrow^{c?} x \supset \nu l_1.(D'[\delta(l_1?Q_1), x] \mid \delta(c_{1,i}?.l_1!))$,
$\nu l_1.(C'[\delta(l_1?P_1), R] \mid \delta(c_{1,i}?.l_1!)) \sim_{c,t} \nu l_1.(D'[\delta(l_1?Q_1), R] \mid \delta(c_{1,i}?.l_1!))$,
for any suitable $R \in CH$. Observe that for any given R there is an

infinite subsequence $\{c_{1,\sigma i}\}_{i<\omega}$ for which the equivalence holds.

(iii) $l_1 \supset D[\delta(l_1?Q_1)] \twoheadrightarrow^{c?} l_1,x \supset D'[\delta(l_1?Q_1), x]$,
$l_1 \supset C'[\delta(l_1?P_1), R]\ S\ D'[\delta(l_1?Q_1), R]$. □

Example

Consider the CH processes: $P_1 \equiv \nu c.\nu d.a!(d?.c?.ø).c!.P'$, $P_2 \equiv a!ø.ø$. We wish to show: $P_1 \sim P_2$ Using the definition of sgb (section 2) a proof of this equivalence reduces to: $\nu c.\nu d.(c!.P' \mid [d?.c?.ø/x]R) \sim [ø/x]R$, for any suitable R. A formal proof of this fact requires some induction on the structure of R and the form of derivations.

On the other hand, in CH_t, we can reduce the problem to:

$\nu c.\nu d.\nu l.a!l.(\delta(l?.d?.c?.ø) \mid c!.P') \sim_{o,t} \nu l.a!l.(\delta(l?.ø)\ )$, i.e.

$\nu c.\nu d.\ (\delta(l?.d?.c?.ø) \mid c!.P') \sim_{o,t} \delta(l?.ø)$,

which can be easily shown by building a suitable ot-sgb. □

4. The π_t-calculus

In this section we introduce a variant of the π-calculus with activation channels to which we reduce the CH_t-calculus with the ot-notion of sgb. We stress that this reduction does not offer new insights on how to reason about CHOCS processes. It is presented here only to complete our trip from CHOCS to the π-calculus. In this respect the interesting point is that, via the translation, one can reason about CH_t processes in the π_t-calculus without losing any valid equivalence.

We distinguish activation and standard channels as for the CH_t-calculus. Activation channels are *the* transmissible values, and, dually, the formal parameter in an input operator denotes an activation channel.[8] The grammar for π_t-processes is:

$$P ::= ø \mid l! \mid (P+P) \mid (P \mid P) \mid (\delta\, P) \mid (\nu c.P) \mid (\nu l.P) \mid (c!l.P) \mid (c?l.\ P) \mid l?.P$$

The rules for the kind judgments are obvious. Again we have six types of actions:

$$\alpha ::= c!l \mid c? \mid \tau \mid l! \mid l? \mid t\ .$$

Lts for π_t

As for CH_t we keep the following rules of the lts in section 2: (?), (+.left/right), (| .left/right), (δ), (ν). We also include the rules for the activation channels: $(!)_t$, $(?)_t$, $(\nu)_t$ and $(\mid.com!?)_t$, as described in section 3. Finally we have the rules (ν.open) and (| .com!?) of section 2 which when specialized to the π_t-calculus go as follows:

$$(\nu.\text{open})\qquad \frac{\Gamma,l\text{: Ch} \supset P \mapsto^{c!l} \Gamma,l\text{: Ch} \supset P'}{\Gamma \supset (\nu l.P) \mapsto^{c!l} \Gamma,l\text{: Ch} \supset P'}$$

[8] In a more general calculus where also standard channels are transmissible values one should design the rules so that no mismatch can arise.

$$(\mathrm{l.com!?})\quad \frac{\Gamma\supset P \mapsto^{c!l} \Gamma,\Gamma'\supset P' \qquad \Gamma\supset Q \mapsto^{c?} \Gamma, x{:}Ch\supset Q'}{\Gamma\supset(P\mid Q)\mapsto^{\tau} \Gamma\supset \nu\Gamma'.(P'\mid [l/x]Q')}$$ [9]

Translation from CH_t to π_t

We define a translation $\langle\ \rangle: CH_t \rightarrow \pi_t$. We suppose to have associated to any process variable x an activation channel l_x.

$\langle c!P'.P\rangle = \nu l.(c!l.\,(\langle P\rangle \mid \delta(l?.\langle P'\rangle))\,)$ for $l\notin P, P'$ $\quad \langle x\rangle = l_x!$
$\langle\emptyset\rangle = \emptyset,\qquad \langle l!\rangle = l!$
$\langle c?x.P\rangle = c?l_x.\langle P\rangle \qquad \langle l?.P\rangle = l?.\langle P\rangle$
$\langle\nu c.P\rangle = \nu c.\langle P\rangle \qquad \langle\nu l.P\rangle = \nu l.\langle P\rangle$
$\langle P\mid P'\rangle = \langle P\rangle\mid\langle P'\rangle \qquad \langle P+P'\rangle = \langle P\rangle+\langle P'\rangle \qquad \langle\delta P\rangle = \delta\langle P\rangle$
$\langle\emptyset\rangle = \emptyset \qquad \langle x{:}Pr,\Gamma\rangle = l_x{:}\ Ch,\langle\Gamma\rangle \qquad \langle x{:}Ch,\Gamma\rangle = x{:}Ch,\langle\Gamma\rangle\ \ (x ::= c \mid l)$
$\langle\Gamma\supset P{:}Pr\rangle = \langle\Gamma\rangle\supset\langle P\rangle{:}Pr$.

CH_t-actions vs. π_t-actions

Given a CH_t process P without free process variables its actions relate to $\langle P\rangle$ actions as follows (the right to left direction can be strengthen in the sense that if $\langle P\rangle$ reduces to Q then *Q is of the form $\langle P'\rangle$* and P reduces to P'):

CH_t		π_t
$\vdash\Gamma\supset P\mapsto^{c!}\Gamma,l{:}Ch\supset P'$	iff	$\vdash\langle\Gamma\rangle\supset\langle P\rangle\mapsto^{c!l}\langle\Gamma\rangle,l{:}Ch\supset\langle P'\rangle$.
$\vdash\Gamma\supset P\mapsto^{c?}\Gamma,x{:}Pr\supset P'$	iff	$\vdash\langle\Gamma\rangle\supset\langle P\rangle\mapsto^{c?}\langle\Gamma\rangle,l_x{:}Ch\supset\langle P'\rangle$.
$\vdash\Gamma\supset P\mapsto^{\alpha}\Gamma\supset P'$	iff	$\vdash\langle\Gamma\rangle\supset\langle P\rangle\mapsto^{\alpha}\langle\Gamma\rangle\supset\langle P'\rangle$ where $\alpha ::= \tau \mid t \mid l! \mid l?$.

We now quickly repeat the development of the calculus along the lines traced for the CH_t-calculus. From the lts '$\mapsto$' for π_t we derive a new lts '$\twoheadrightarrow$' applying the definition in section 3.

Sgb for π_t

Next on top of the lts '$\mapsto$' and '$\twoheadrightarrow$' we build a notion of sgb which corresponds to the simplified definition for the π-calculus we have presented in section 2.

Definition *(sgb for π_t-calculus)*

A relation S is a π-*sgb* if $\Gamma\supset P\ S\ Q$ and for any $\vdash\Gamma\supset P\mapsto^{\alpha}\Gamma'\supset P'$ then:

$\alpha\equiv\tau, t, l?, l!$ implies there is $\vdash\Gamma\supset Q\mapsto^{\alpha}\Gamma\supset Q'$, $\Gamma\supset P'SQ'$.

$\alpha\equiv c?$, $\Gamma'\equiv\Gamma,l{:}Ch$, implies there is

$\vdash\Gamma\supset Q\mapsto^{c?}\Gamma,l{:}Ch\supset Q'$, $\Gamma,\underline{l{:}Ch}\supset P'\ S\ Q'$, [10]

$\alpha\equiv c!l$ implies there is $\vdash\Gamma\supset Q\mapsto^{c!l}\Gamma,\Gamma'\supset Q'$, and $\Gamma,\Gamma'\supset P'SQ'$.

and symmetrically.

[9] Γ' can be either empty if there is no scope extrusion on l, or $\Gamma'\equiv l{:}Ch$ o.w. .

[10] The 'underline' notation of strong bisimulation is used here.

We write $\Gamma \supset P \sim\pi\ Q$ if $\Gamma \supset P\ S\ Q$ for some S π-sgb. If we substitute everywhere the relation $\mapsto$ with the relation $\twoheadrightarrow$ we obtain the notion of *πt-sgb*. The greatest of these sgbs is denoted with $\sim\pi t$.

Lemma *(translation and substitution)*

Suppose $\vdash\Gamma,\Gamma'',x{:}Pr\supset R{:}Pr$, $\vdash\Gamma,\Gamma'\supset P{:}Pr$, where Γ,Γ',Γ'' is well-formed, and P, R are CH_t-processes. Then: $\underline{\Gamma,\Gamma',\Gamma''}\supset\ <[P/x]R> \sim\pi t <\{P/x\}R>$.

Proof

The proof is basically the same as for the CH_t calculus. □

The two following propositions tie up *ot-sgb* and *πt-sgb*.

Proposition *($\sim_{\pi,t}$ implies $\sim_{o,t}$)*

Suppose P and Q are CH_t-processes in the context Γ. Then

$$<\Gamma>\supset <P> \sim_{\pi,t} <Q> \ \Rightarrow\ \Gamma\supset P \sim_{o,t} Q\,.$$

Proof

We define: $\Gamma\supset P\ S\ Q$ if $<\Gamma>\supset <P> \sim_{\pi,t} <Q>$. We show S is an ot-sgb by using the correspondence between the actions. The only interesting case is when we make and input action on a standard channel. Then one can reason as follows:

(?) (i) $P\ S\ Q,\ P \xrightarrow{c?\ x}\!\!\supset P'$

(ii) $<P> \sim_{\pi,t} <Q>,\ <P> \xrightarrow{c?\ l_x}\!\!\supset <P'>,$
$<Q> \xrightarrow{c?\ l_x}\!\!\supset <Q'>,\ \underline{l_x}\supset\ <P'> \sim_{\pi,t} <Q'>$, which implies:
$\nu l_x.(<P'> \mid \delta(l_x?.<R>)) \sim_{\pi,t} \nu l_x.(<Q'> \mid \delta(l_x?.<R>))$ any R in CH. Observe:
$\nu l_x.(<P'> \mid \delta(l_x?.<R>)) \equiv <\{R/x\}P> \sim_{\pi,t} <[R/x]P>$ (sub. lemma).

(iii) $Q \xrightarrow{c?\ x}\!\!\supset Q',\ [R/x]P'\ S\ [R/x]Q'$, any R in CH. □

Proposition *($\sim_{o,t}$ implies $\sim_{\pi,t}$)*

Suppose P and Q are CH_t-processes in the context Γ with no free process variables. Then

$$\Gamma\supset P \sim_{o,t} Q \ \Rightarrow\ <\Gamma>\supset <P> \sim_{\pi,t} <Q>.$$

Proof

The side condition is not a real limitation as in general $\Gamma,\underline{x{:}Pr} \supset P \sim Q$ is equivalent to $\Gamma,c{:}Ch \supset c?x.P \sim c?x.Q$. We define: $<\Gamma>\supset <P>\ S\ <Q>$ if $\Gamma\supset P \sim_{o,t} Q$. We show S is an ot-sgb by using the correspondence between the actions. Again the input case is the interesting one.

(?) (i) $<P>\ S\ <Q>,\ <P> \xrightarrow{c?\ l_x}\!\!\supset <P'>$.

(ii) $P \sim_{o,t} Q,\ P \xrightarrow{c?\ x}\!\!\supset P',\ Q \xrightarrow{c?\ x}\!\!\supset Q'$,
$[R/x]P' \sim_{o,t} [R/x]Q'$, any R in CH. By the input lemma
$[l_x!/x]P' \sim_{o,t} [l_x!/x]Q'$. Also observe for any l (possibly in P', Q') we have:
$[l!/x]P' \sim_{o,t} \nu l_x.([l_x!/x]P' \mid \delta(l_x?.l!)) \sim_{o,t} \nu l_x.([l_x!/x]Q' \mid \delta(l_x?.l!)) \sim_{o,t} [l!/x]Q'$.

(iii) $<Q> \xrightarrow{c?\ l_x}\!\!\supset <Q'>,\ [l/l_x]<P'>\ S\ [l/l_x]<P'>$, any l. □

5. The Reduction

We can now collect our results and make some further remark on their application:

Theorem *(reduction)*

Suppose P and Q are CH-processes in the context Γ with no free process variables. Then

$$\Gamma \supset P \sim Q \Leftrightarrow \langle\Gamma\rangle \supset \langle P\rangle \sim_{\pi,t} \langle Q\rangle.$$

Proof

We have shown the following chain of equivalences:

$$\Gamma \supset P \sim Q \Leftrightarrow^{(1)} \Gamma \supset P \sim_{c,t} Q \Leftrightarrow^{(2)} \Gamma \supset P \sim_{o,t} Q \Leftrightarrow^{(3)} \langle\Gamma\rangle \supset \langle P\rangle \sim_{\pi,t} \langle Q\rangle.$$

A rough comment of the significance of each step goes as follows: (1) gives a conceptual simplification of the output action for CHOCS by reducing the sending of a process to the sending of an activation channel. Using (generic input) one also observes the finiteness of the (?) clause of the sgb. (2) makes the (!) clause of the sgb *local*, by eliminating any reference to the contextual process R. (3) shows that the additional checks needed in the (?) clause of π-calculus sgb do not induce a finer equivalence on CH_t processes. □

Concerning the computability of the reduction relation '$\twoheadrightarrow$' one will remark the following

Proposition *(on computability of $\twoheadrightarrow$)*

Suppose $\vdash \Gamma \supset P: Pr$ in CHOCS where P has no δ.[11] Then for any derivative:

$$\langle\Gamma\rangle \supset \langle P\rangle \twoheadrightarrow^{\alpha_1} \Gamma_1 \supset P_1 \ldots \twoheadrightarrow^{\alpha_n} \Gamma_n \supset P_n \qquad (n \geq 1)$$

in the π_t-calculus, the collection $\{P' \mid \Gamma_n \supset P_n \ (\mapsto^t)^* \ \Gamma_n \supset P'\}$ is finite and computable.

Proof

First consider the process $c?x.(c!x \mid c?y.\ (\delta y)\ )$. The translation gives something equivalent to: $c?l_x.(\nu l.c!l.\ \delta(l?l_x!) \mid c?l_y.\delta(l_y!)\ ) \twoheadrightarrow^{c?} \ldots \twoheadrightarrow^{\tau} \delta(l?l_x!) \mid \delta(l!)\ (\mapsto^t)^\omega$, which shows the need for excluding duplication in P. Also we do not consider the case n=0 as $\langle P\rangle$ cannot perform t transitions.

Next we observe that derivatives of $\langle P\rangle$ can be seen, up to equivalence, as processes with the following structure:

$$\nu l_1 \ldots \nu l_n.C[\delta(l_1?.\langle P_1\rangle), \ldots, \delta(l_n?.\langle P_n\rangle), l_1!, \ldots, l_1!, \ldots, l_n!, \ldots, l_n!]\ .$$

with the convention that the only occurrences of the bound activation channels l_i are the ones displayed. The only possibility to perform a t transition is to synchronize on some activation channel l_i . This can be repeated as many times as there are activators $l_1!, \ldots, l_1!, \ldots, l_n!, \ldots, l_n!$. After that, one is forced to do a distinct action because the processes $\langle P_i\rangle$ which one has been duplicating cannot contribute to a t action right away, by the form of the translation. □

[11] To simulate duplication set (after Thomsen): $\delta_{CH}(P) \equiv \nu c.(D(c, P) \mid c!D(c, P)\ )$, where $D(c, P) \equiv c?x.(P \mid x \mid c!x)$, $x, c \notin P$. Then: $\delta_{CH}(P) \mapsto^{\tau} \sim P \mid \delta_{CH}(P)$.

Comparison with related work

Sangiorgi (Sangiorgi[92]) has indipendently obtained related results which now we try to frame in our context. Sangiorgi considers a 'higher-order π-calculus' which includes CHOCS as a particular case. For this calculus a notion of 'context bisimulation' is introduced, which on CHOCS processes appears to be the same as the notion of sgb introduced in section 2 (actually all Sangiorgi's work takes place in the context of early binding, but as we mentioned this distinction is not relevant for CHOCS processes). His next step is to introduce a notion of 'trigger bisimulation' with the declared goal of reducing the complexity of reasoning on 'context bisimulation'. The notion of 'trigger' appears directly connected to our idea of activation channel, and hence to Thomsen's translation of CHOCS into π-calculus. It is worth emphasizing that Sangiorgi restricts the calculus by only allowing 'guarded sums'. On one hand this restriction allows to keep the 'standard' π-calculus as target of the CHOCS translation, on the other hand his technique cannot deal with the simple and natural CHOCS equivalence discussed in the introduction.

6. Conclusion

We have reduced the complexity of reasoning about CHOCS sgb. We expect that our method can be extended to other equivalences such as weak bisimulation. At any rate the translation into the π_t-calculus is so natural that one could consider it as *the* semantics of CHOCS in terms of a more fundamental calculus.

The translation we used associates a duplication to every output. This means that in most interesting cases the process <P> associated to the CHOCS process P will have an infinite number of derivatives. This rises the challenging problem of developing effective tools to reason about π-calculus processes with duplication.

In the lts for CH_t actions are really labels as in CCS. We expect that this point will simplify the definition of a denotational semantics for CHOCS.

Acknowledgments

The idea of distinguishing two types of internal actions first arose in discussions with Gérard Berry and the Meije group in Sophia-Antipolis in June '92. This idea remained unexploited for sometimes as its formalization and utility were unclear. A month later its role became obvious to me in the example given in the introduction, then a complete formalization shortly followed.

References

Amadio R. [1992] "A uniform presentation of CHOCS and π-calculus", RR INRIA-Lorraine, 1726, 35 pages.

Astesiano E., Zucca E. [1984] "Parametric channels via Label Expressions in CCS", TCS, 33, (45-64).

Boudol G. [1989] "Towards a lambda calculus for concurrent and communicating systems", in Proc. TAPSOFT, SLNCS 351.

Engberg U., Nielsen M. [1986] "A calculus of communicating systems with label passing", DAIMI PB-208, CS Dept. Aarhus.

Milner R. [1989] "Communication and Concurrency", Prentice Hall.

Milner R., Parrow J., Walker D. [1989] "A calculus of mobile processes I, II", Info.&Comp., 100, 1, (1-77), 1992. Also appeared as ECS-LFCS TR 89-85, 89-86, Edinburgh.

Nielsen M. [1988] "The typed lambda calculus with first class processes", in Proc. PARLE 89, SLNCS 366, also appeared as TR 1988-43, Inst. for Datateknik, Lingby.

Sangiorgi D. [1992] "Expressing mobility in process algebras: first-order and higher order paradigms", draft PhD thesis, University of Edinburgh, September 1992.

Thomsen B. [1989] "A calculus of higher order communicating systems", in Proc. ACM-POPL, 1989.

Thomsen B. [1989(a)] "Plain CHOCS", TR 89/4, DOC Imperial College, London, 1989.

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